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# CHAOS VIA TORUS DESTRUCTION IN MODELS OF DENGUE FEVER AND PREDATOR-PREY SYSTEMS, IMPLICATIONS FOR DATA ANALYSIS

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In the analysis of multi-strain models describing dengue fever epidemiology we found complex bifurcation structures, and especially the appearance of deterministic chaos after torus bifurcations [1]. Since the fluctuations of severe dengue fever cases e.g. in Thailand and its provinces can be well described by such models, the next step is to investigate algorithms for time series analysis even under deterministic chaos, see e.g. [2] and references therein. Based on dynamic noise [3] one can obtain estimations of likelihood functions and apply the whole toolbox of parameter estimation and model evaluation in principle, however still under technical difficulties of long computer runs.

To understand the transition into chaos after torus bifurcations better we therefore searched for simpler population models than the already quite high dimensional dengue models. One of the best candidates is the in itself two-dimensional seasonal Rosenzweig-MacArthur model which was described to undergo torus bifurcations, e.g. in [4] and [5], where also regions of deterministic chaos were found by increasing parameters crossing torus bifurcations. The chaotic regions could only be speculated and exemplified by individual simulations but their parameter regions could only be guessed fuzzily. We investigated these models again and now not only with AUTO to detect bifurcation structures, but also with Lyapunov spectra in which the same bifurcation lines could

be detected by looking at subdominant Lyapunov exponents reaching zero and also chaotic parameter regions could be detected. Surprisingly, the previously only fuzzy guessed chaotic regions turned out to be part of Arnol'd tongues on the tori after the torus bifucations. For the analysis of empirical data, of course stochastic versions of the models have to be investigated, in the case of the Rosenzweig-MacArthur model time scale separable stoichiometric versions. The results are summarized in [6].

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